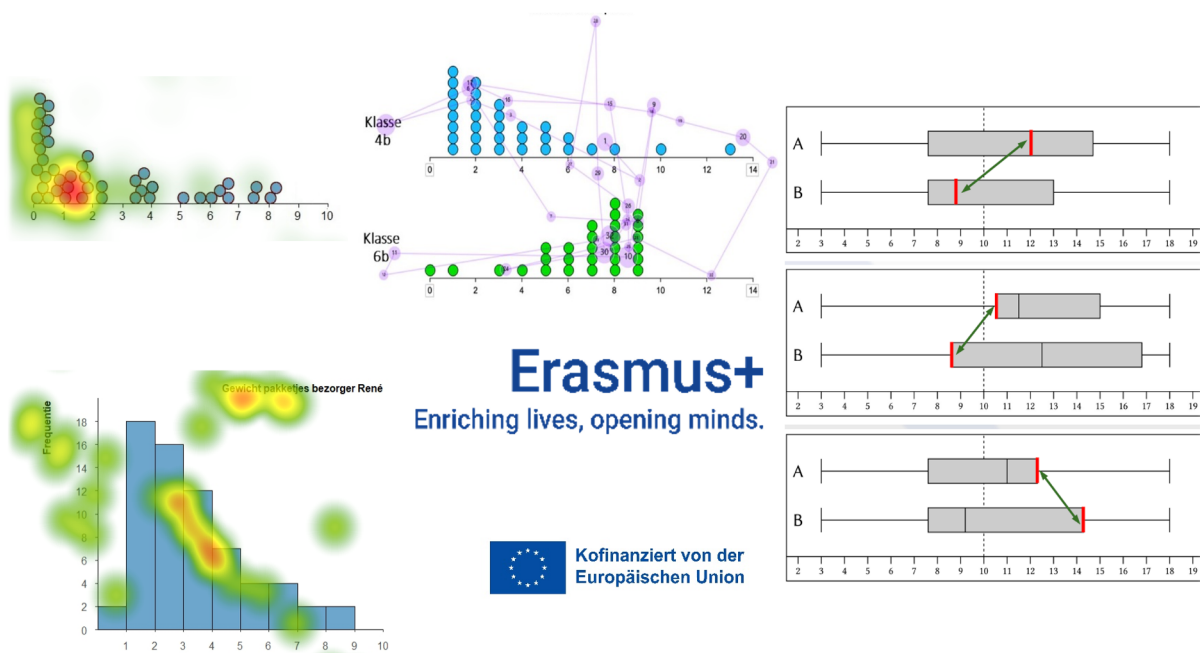


EYE-TEACH-STATS

Course Manual

A guide for teacher educators implementing the eye-tracking vignette course on students' statistical graph interpretation.



Erasmus+ Programme – Eye-Teach-Stats Project

www.eye-teach-stats.eu

Disclaimer:

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How to Use This Manual

This manual is intended for teacher educators who implement the Eye-Teach-Stats course with pre- and in-service mathematics teachers or student teachers. It provides the theoretical and pedagogical background for the course materials and offers practical guidance for facilitating the five modules in the course. Throughout this manual, teachers refers to the pre-service teachers, in-service teachers, or student teachers participating in the course, while students refers to the primary or secondary school pupils taught by these teachers.

Who is this manual for?

This manual is written for teacher educators—those who facilitate the course, guide discussions, and support participants as they work through the eye-tracking vignettes. It is not intended as a participant handout.

How is this manual structured?

The manual is organised into six sections:

- **Section 1** introduces the course and its aims.
- **Section 2** provides the theoretical background, covering teachers' professional competence, diagnostic activities, systematic errors, and the role of eye-tracking evidence.
- **Section 3** explains the eye-tracking vignette format and how it is structured as a learning task.
- **Section 4** describes each of the four statistical graph types in the course, including the systematic errors addressed in the vignettes.
- **Section 5** outlines the instructional design of the course, based on the 4C/ID model.
- **Section 6** provides practical guidance for implementing the course, including how to facilitate face-to-face or online synchronous sessions, individual work, and reflection activities.

How to use this manual alongside the course materials

This manual is designed to be used together with:

- **The online learning environment (MaMpf)**, in which participants access and complete eye-tracking vignettes.
- **The Teaching Modules slide deck** is used during face-to-face or online synchronous sessions. It provides the course introduction, module overviews, and discussion slides that structure group conversations after participants have worked through the vignettes in MaMpf. In the remainder of this manual, these are referred to as synchronous group sessions.
- **This manual** provides teacher educators with the theoretical and pedagogical background for the course and guidance on facilitating sessions.

The discussion slides in the participant slide deck each contain notes for teacher educators. These notes include guiding questions tailored to the specific vignette and error type on each slide. We suggest referring to these notes when facilitating group discussions. Section 6.6 of



this manual describes the purposes of these guiding questions and provides examples of how they can be used.

1. Introduction

This course is designed to support pre- and in-service mathematics teachers in working with students' statistical (mis)interpretations when interpreting graphs. It focuses on four graph types, dotplots, histograms, boxplots, and case-value plots, and aims to contribute to the development of teachers' statistical content knowledge (CK) and pedagogical content knowledge (PCK).

A central element of the course is the use of eye-tracking vignettes. These vignettes are based on research about students' typical errors and conceptual difficulties when interpreting graphs and are used throughout the five modules (introduction to eye-tracking and one module per graph type).

The course can be implemented in both pre-service teacher education and professional development settings. This manual provides teacher educators with background information on the course concept and materials, as well as guidance for supporting participants as they work through the modules.

Eye-Teach-Stats is an Erasmus+-funded international project developed by five partner institutions (see About the Eye-Teach-Stats Project at the end of this manual for full details).

2. Theoretical Background

The theoretical background of the course is organised around the relation between teachers' professional competence, diagnostic reasoning, students' systematic errors, and the use of eye-tracking evidence. Figure 1 provides an overview of this relationship by showing how diagnostic work is shaped by the characteristics of the situation, the teacher's knowledge and beliefs, and the processes of perceiving, interpreting, and decision-making. In this course, eye-tracking vignettes are used to make these processes explicit: teachers examine students' answers alongside gaze data and use this evidence to form hypotheses about students' interpretations of graphs.

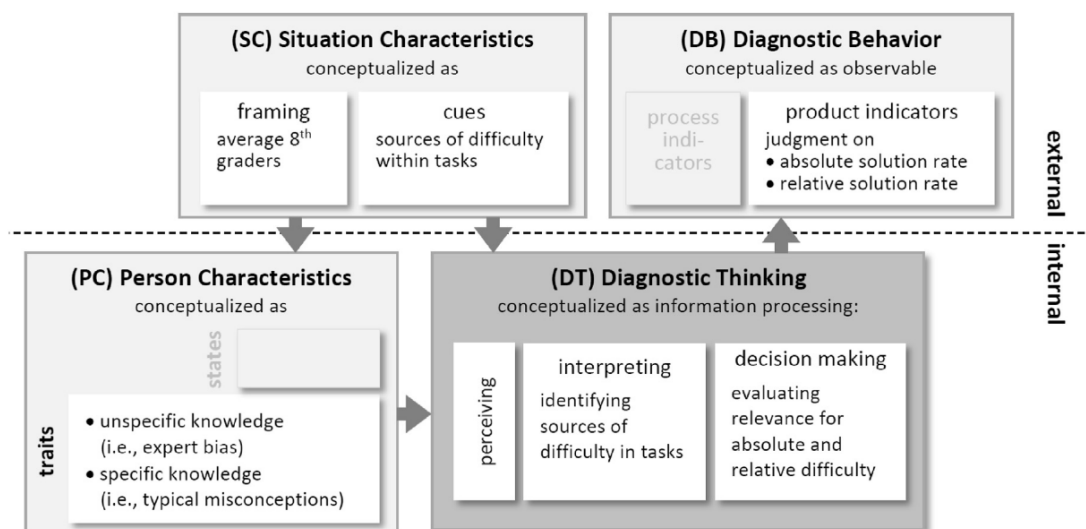


Figure 1. Overview of diagnostic activities and the factors that shape them (Loibl et al., 2020,p.3)

2.1 Professional Competence

Figure 2 situates the course within a broader model of teachers' professional competence. In the COACTIV model, professional competence comprises several components, including professional knowledge, beliefs, motivation, and self-regulation. The Eye-Teach-Stats course focuses particularly on two facets of professional knowledge that are central for teaching statistics: content knowledge (CK) and pedagogical content knowledge (PCK).

In this context, content knowledge refers to teachers' understanding of the properties and interpretations of different statistical graphs, whereas pedagogical content knowledge refers to teachers' ability to identify students' interpretations and misinterpretations and relate them to underlying conceptual difficulties. Both knowledge facets are addressed throughout the course by asking teachers to analyse students' responses and eye-tracking data in relation to dotplots, histograms, boxplots, and case-value plots.

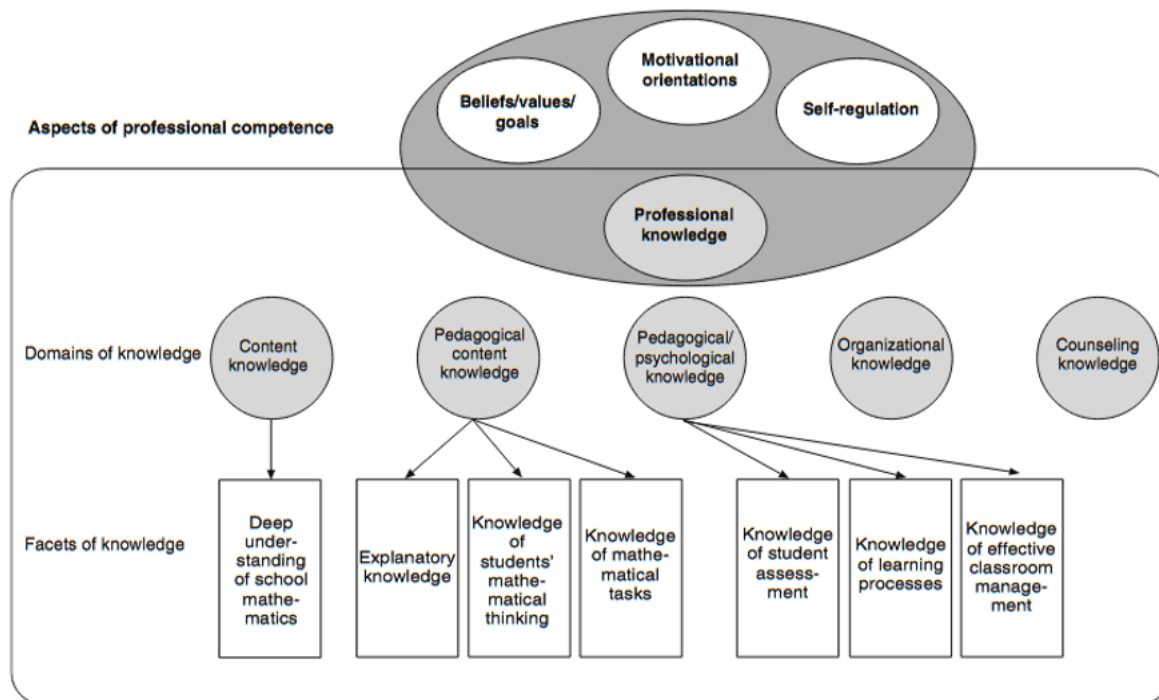


Figure 2. Model of teachers' professional competence (COACTIV model). [Baumert & Kunter, 2013,p.29]

2.2 Diagnostic Activities and Diagnostic Competence

Diagnostic activities are those in which teachers gather and interpret information about students' thinking to make pedagogical decisions. These activities include, for example, listening to students' explanations, analysing written work, and comparing different solution strategies.

Vignettes are considered particularly suitable for such diagnostic activities because they can capture the complexity of students' reasoning in a concrete situation. Eye-tracking vignettes extend this idea by adding information about students' visual behaviour while they work on a task.

The set of teachers' knowledge, skills, motivations, and beliefs that enables them to carry out diagnostic activities purposefully is referred to as diagnostic competence. In the context of this course, diagnostic competence focuses on identifying how students interpret graphs, and recognising typical (mis)interpretations.

2.3 Systematic Errors and Underlying Conceptual Difficulties

When interpreting graphs, students often make systematic errors: recurring patterns of reasoning that lead to similar incorrect conclusions across tasks or graph types. Systematic errors are closely connected to underlying conceptual difficulties. While a systematic error describes how a student reasons incorrectly in a particular representation, the conceptual difficulty reflects what is not yet fully understood—for instance, the idea of distribution, the meaning of the mean, or the role of variability. In this course, systematic errors in interpreting dotplots, histograms, boxplots, and case-value plots are used as entry points to reason about these deeper conceptual issues in students' understanding.

In the working framework developed for this project, a full set of systematic errors was identified for each of the four graph types through a systematic literature review (Heursen et al., 2025). Table 1 provides an overview of these systematic errors, as represented in the eye-tracking vignettes, along with brief definitions.

Table 1. Overview of systematic errors across graph types.

Category	Systematic Error	Definition
Graph mechanics representation confusion	Frequency–value confusion	Incorrect estimation of the mean of frequencies instead of the mean value.
	Area bias	Incorrect assumption that a longer box represents a higher frequency of observations.
	Overgeneralisation of a histogram approach	Incorrect assumption that a case-value plot can be interpreted like a histogram, based on shape or balance rather than on individual measured values
Overemphasising local features	Pairwise-stack comparison	Judgement based on comparing corresponding stacks/bars one by one, rather than on the distribution as a whole.
	Overemphasising outlier	Judgement based on an outlier rather than on the distribution.
	Overemphasising number of bars	Judgement based on the number of bars rather than on the distribution.
	Count-and-compute strategy	Bars are processed one by one rather than as a distribution, leading to dividing by the number of bars instead of the total number of data points.
Looking at a wrong parameter for measures of central tendency	Sum for mean: Sum of frequencies	Frequencies are added up to draw conclusions about statistical concepts or the overall data distribution.
	Sum for mean: Sum of measured values	Data values are added up—multiplied by their frequencies—to draw conclusions about the distribution.
	Median bias	Judgement based on an incorrect measure of central tendency.

2.4 Eye-Tracking as Evidence for Student Reasoning

Eye-tracking technology records and maps students' eye movements while they work on a task. This provides information about where students look, in what order, and for how long. Such visual behaviour can be related to the strategies students use and the aspects of the

graph they consider relevant. In this course, eye-tracking is used as an additional source of evidence alongside students' written answers and verbal explanations.

Three main types of eye-tracking visualisations are used in the course:

- **Gaze plots.** Static images that display the sequence of eye movements on the graph. Fixations are represented as dots; their size reflects how long the student looked at that location. Lines between dots (saccades) show how attention shifted across the graph.



Figure 3. Example of a gaze plot.

- **Gaze videos.** Dynamic visualisations that replay the student's eye movements over time on top of the original task, making the temporal order of inspection visible.
- **Heatmaps.** Colour-coded overviews indicating where the student's gaze was concentrated. Warmer colours show areas of longer or more frequent fixations; cooler colours indicate less attention.

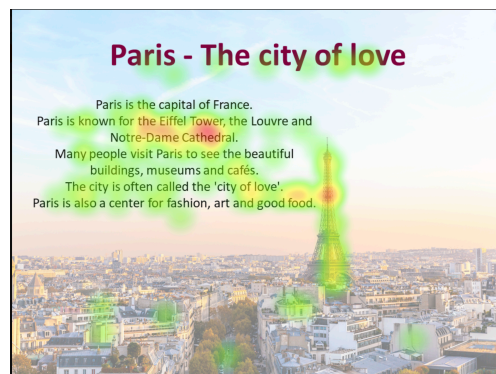


Figure 4. Example of a heatmap.

These visualisations make aspects of students' visual search strategies visible that might remain hidden in written or spoken responses alone. When combined with the student's answer, they provide a richer basis for forming and justifying hypotheses about students' interpretations of graphs and for identifying systematic errors and underlying conceptual difficulties.

3. Eye-Tracking Vignettes

This section describes how eye-tracking vignettes are defined in the course, how they are structured as learning tasks, and why this format is particularly suitable for supporting teachers' analysis of students' graph interpretations.

3.1 What are Vignettes?

Vignettes present short, authentic scenes from teaching practice. They allow teachers to closely examine and reflect on classroom situations without the pressure to act immediately. By focusing on specific aspects of a situation, vignettes enable structured, meaningful discussion of teaching and learning.

In mathematics teacher education, vignettes are used to connect theory to classroom practice. They support professional learning by encouraging teachers to analyse students' responses, reasoning, or errors, and to reflect on instructional decisions in a safe and focused setting. Vignettes can be designed in different ways—as texts, videos, or visual representations—and can invite open responses or more structured answers. Depending on the task, in general, vignettes may ask teachers to interpret a situation, evaluate students' thinking, or consider possible underlying conceptual difficulties students might have.

3.2 What are Eye-Tracking Vignettes?

In this course, eye-tracking vignettes are used as an innovative format that supports teachers in analysing students' interpretations of statistical graphs. Each vignette presents a short scenario in which a student works on a graph interpretation task.

Each eye-tracking vignette combines:

- the graph task,
- the student's answer (often with a short explanation), and
- a visualisation of the student's eye movements while working on the task.

The vignettes are based on real student data and highlight systematic errors in interpreting dotplots, histograms, boxplots, and case-value plots. By showing both the student's answer and visual attention, they provide insight into not only what the student concluded, but also how the student approached the graph. Each vignette includes guiding questions that invite teachers to analyse the student's reasoning, identify possible systematic errors, and relate these to the graph type and task context.

3.3 Structure of an Eye-Tracking Vignette

Within the course, each eye-tracking vignette follows a recurring structure (see Figure 5). It begins by introducing a classroom scenario and presenting the statistical graph task. Participants then work through a sequence in which they examine the student's answer, eye-tracking data, and retrospective explanation. After this sequence, they receive an explanation of the systematic error and its possible underlying conceptual difficulty and complete a related part-task practice activity. Supportive and procedural information is available at relevant points throughout the vignette. At the core of each vignette, participants work through the following steps:

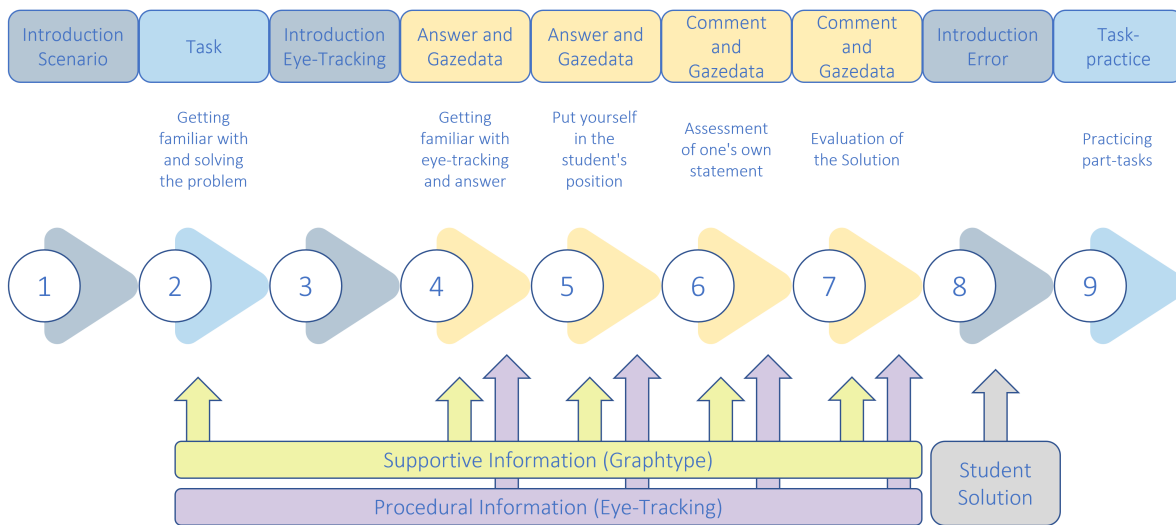


Figure 5. Structure of an eye-tracking vignette.

3.4 Diagnostic Potential of Eye-Tracking Vignettes

Eye-tracking vignettes extend traditional vignettes by adding visual information about how a student engaged with a graph. This allows teachers to:

- link systematic errors to specific features of the graph that were attended to or ignored;
- distinguish difficulties that stem from not noticing information from those that stem from misinterpreting it;
- justify conclusions with reference to both verbal and visual evidence.

In this way, eye-tracking vignettes give teachers access to aspects of students' visual attention that are normally not visible. For example, they can reveal whether a student attended to relevant information but did not use it or interpreted it differently from what the graph required.

4. Statistical Graphs in the Course

This course focuses on four types of statistical graphs: dotplots, histograms, case-value plots, and boxplots. Each graph type offers specific opportunities for reasoning about distributions and is associated with typical systematic errors in students' interpretations. This section provides brief descriptions of the four graph types and summarises the systematic errors central to the course.

The eye-tracking vignettes in this course are developed based on findings from recent research (e.g., Abt et al., 2023, 2024, 2025; Boels et al., 2019a, 2019b, 2023a, 2023b, 2024, 2025; Schreiter & Vogel, 2022, 2023a, 2023b, 2024). These studies have demonstrated that students' systematic errors and underlying conceptual difficulties are reflected in their gaze data. While the literature identifies a broader range of errors (summarised in Section 2.3), the vignettes focus on a specific subset for each graph type.

4.1 Dotplots

Description

Dotplots are simple yet effective graphical representations that display the distribution of a dataset. Each data point is represented by a dot placed along a single axis. When multiple data points share the same value, their dots are stacked vertically, making frequency patterns immediately visible. Dotplots are particularly useful for small to medium-sized datasets and serve as an introduction to data visualisation in statistics education.

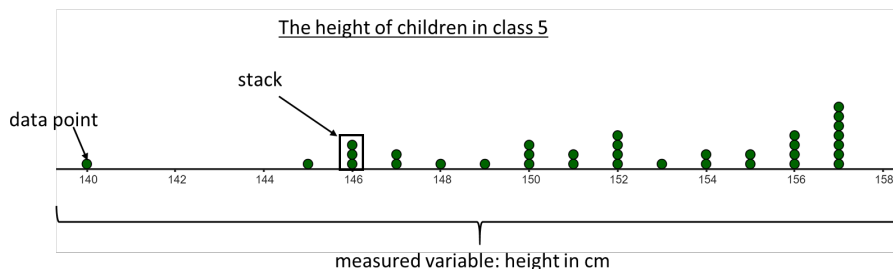


Figure 6. Example of a dotplot.

Local vs. Global Perspectives

A fundamental goal in teaching statistics is to help students move beyond a “local view” of data distributions—where they focus on individual data points—and develop a “global view” that allows them to recognise overarching trends (Schreiter & Vogel, 2023).

- **Local view.** Students may perceive a distribution as a collection of isolated data points, focusing on specific values and extreme cases rather than considering the dataset as a whole. This approach can lead to misconceptions about variability and central tendencies.
- **Global view.** A more advanced perspective encourages students to interpret distributions holistically, identifying key statistical features such as centre (mean, median, mode), spread (range, interquartile range, standard deviation), and shape (symmetry, skewness, modality). While the graphical appearance of distributional shape depends on the type of representation, characteristics such as skewness,

modality, or tail behaviour remain global properties of the underlying distribution. Their visual manifestation therefore needs to be interpreted in relation to the specific graphical representation.

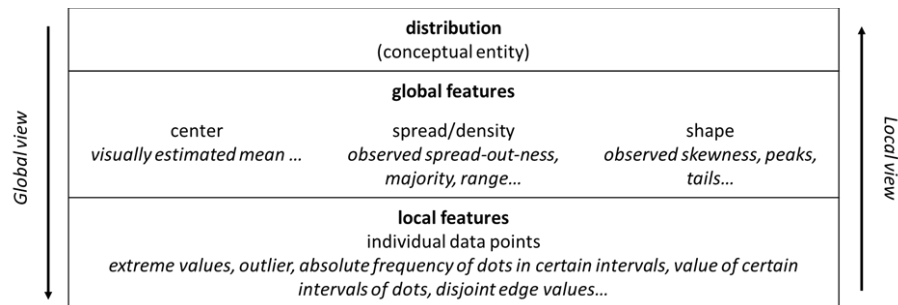


Figure 7. Local and global perspectives on data distributions (Schreiter & Vogel, 2023).

When analysing dotplots, students should be encouraged to consider both global and local characteristics. By guiding students to compare dotplots with an emphasis on both perspectives, educators can foster a deeper understanding of statistics and critical thinking skills.

For dotplots, the eye-tracking vignettes focus on the following systematic errors:

Systematic error	Definition
Frequency–value confusion	Incorrect estimation of the mean of frequencies instead of the mean value.
Overemphasising outlier	Judgement based on an outlier rather than on the distribution.
Pairwise stack comparison	Judgement based on comparing corresponding stacks/bars one by one, rather than on the distribution as a whole.
Sum of measured values	Data values are added up—multiplied by their frequencies—to draw conclusions about the distribution.

4.2 Histograms

Description

A histogram is an essential graph in research, education, and everyday media. It displays data using bars, where each bar represents a range of values (a bin) rather than individual data points. Unlike measures of central tendency, which summarise data with a single value, histograms show how data is spread across different intervals, making patterns and trends easier to recognise. They are particularly suitable for large datasets of continuous numerical data.

Key characteristics of histograms:

- Each bar represents a range of values (a bin). The horizontal axis represents the variable of interest; the vertical axis shows frequency, relative frequency, or class density.
- When bin widths are equal, bar height corresponds directly to frequency. When bin widths differ, class density is used to ensure accurate representation.
- The total relative frequency sums to 100%, or the total density sums to 1.

To illustrate: consider visualising the ages of 826,192 COVID-19 infections reported in the Netherlands in 2020 (RIVM, 2022). Plotting each individual data point would be unmanageable. By grouping ages into 10-year intervals, a histogram clearly represents the distribution with just a few bars, making it easy to identify which age groups were most affected.

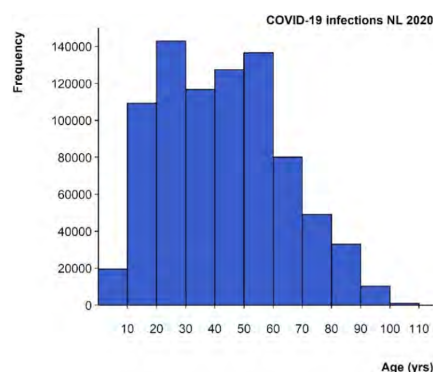


Figure 8. Frequency histogram for all 826,192 COVID-19 infections in the Netherlands in 2020 (Boels, 2023)

Frequency and Density Histograms

Histograms can display data in two main ways:

- **Frequency histograms** (Figure 8) show raw counts in each interval, useful when the focus is on the total number of observations in each group.
- **Density histograms** (Figure 9) display density per unit on the horizontal axis, so that the total area of all bars sums to 1. This allows meaningful comparison across datasets of different sizes or with unequal bin widths (Freedman et al., 1978).

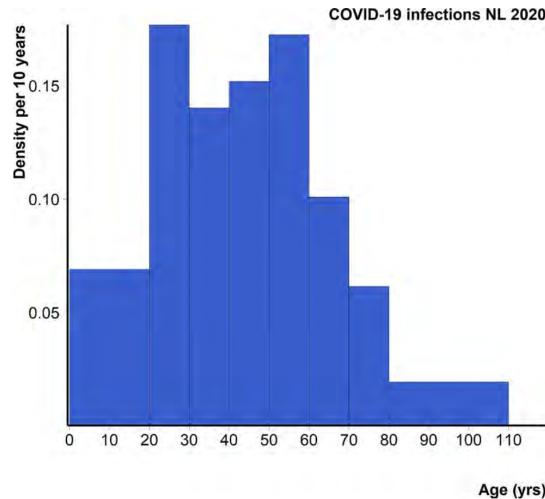


Figure 9. Density histogram for COVID-19 infections per 10-year interval in the Netherlands in 2020 (Boels, 2023).

The Mean in a Histogram

The mean age of infection in the COVID-19 example is approximately 43 years. It can be estimated from a histogram by multiplying each bin midpoint by its corresponding bar height (frequency), summing the products, and dividing by the total number of observations. This value can be marked on both histogram types as a vertical line at 43 on the horizontal axis, providing a central reference point for reading the distribution.

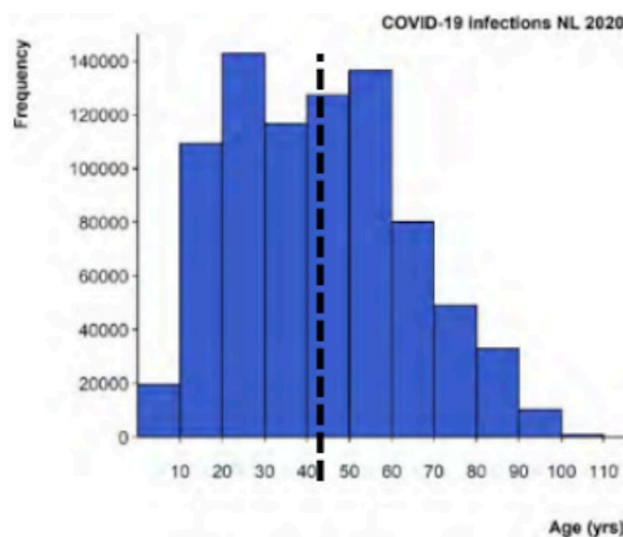


Figure 10. Frequency histogram for COVID-19 infections in the Netherlands in 2020, with the mean age (43 years) marked as a vertical line.

Local vs. Global Perspectives

As with dotplots, a key learning goal is for students to move from interpreting individual bars (local view) to recognising the overall shape and density of the distribution (global view). In histograms, this shift is particularly important because students must interpret aggregated data rather than individual values (Schreiter & Vogel, 2023; see Figure 7 in the dotplot section).

For histograms, the eye-tracking vignettes focus on the following systematic errors:

Systematic error	Definition
Frequency–value confusion	Incorrect estimation of the mean of frequencies instead of the mean value.
Sum of frequencies	Frequencies are added up—often within specific intervals—to draw conclusions about statistical concepts or the overall data distribution.
Pairwise stack comparison	Judgement based on comparing corresponding stacks/bars one by one, rather than on the distribution as a whole.
Overemphasising number of bars	Judgement based on the number of bars rather than on the distribution.

4.3 Case-Value Plots

Description

A case-value plot (CVP) is a graph with bars where each bar's height represents a single measured value. The horizontal axis displays nominal categories, while the vertical axis represents numerical values. Since each bar corresponds to an individual measured value rather than grouped frequencies, variation can be observed as differences in bar heights across categories.

Unlike histograms, which group data into bins to show frequency distributions, case-value plots focus on individual measured values. This makes them useful for visually comparing values across different groups or categories based on individual data rather than analysing the shape of a distribution.

Example: Ages of Teachers in a Department

Consider the case-value plot that shows the ages of the eight teachers in the mathematics department of a school. Each bar represents one teacher. The mean age can be represented by a horizontal line at 40 years, crossing the vertical axis at that number.

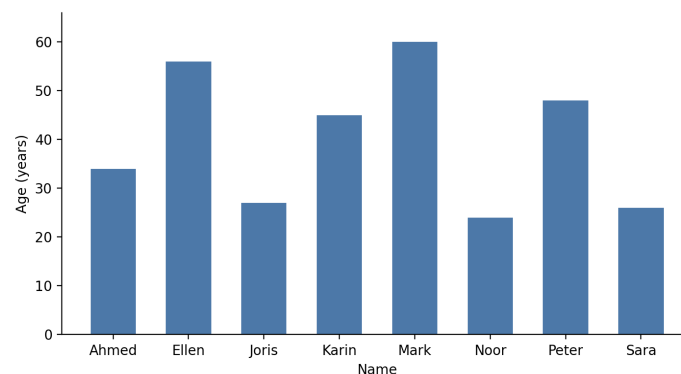


Figure 11. Age (in years) of the eight teachers in a mathematics department.

This example shows how a case-value plot displays the measured value of each individual teacher, making it easy to compare the teachers' ages. The horizontal line at 40 represents the mean of all measured values. Students can visually estimate where the mean lies from the heights of the bars by, as it were, making all bars equally high: what rises above the line fills in what is missing below it (see Figure 12).

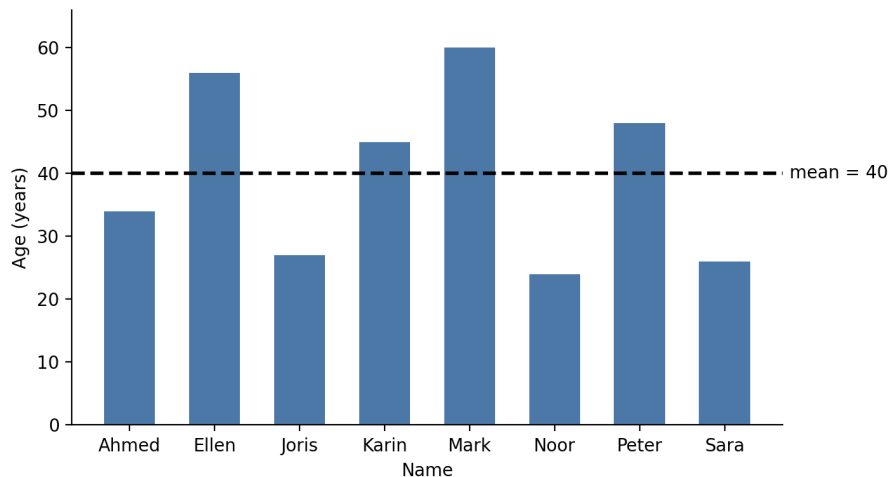


Figure 12. Age (in years) of the eight teachers in a mathematics department, with the mean shown as a horizontal line.

Distinguishing Case-Value Plots from Histograms

Students often confuse case-value plots with histograms, as both use bars. However, not every graph with bars is a histogram. The key differences are:

- The horizontal axis in a case-value plot represents a nominal or ordinal variable (e.g., the teachers' names in Figure 12). A mean of names has no meaning. In a histogram, the horizontal axis represents a continuous numerical variable, and the mean (e.g., a mean age of 43 years) is a meaningful value on that axis.
- In a case-value plot, the height of a bar is the measured value of one case (e.g., the age of one teacher). In a histogram, the height of a bar is the number of measurements within an interval.
- In a case-value plot, the mean of the individual measured values is shown as a horizontal line crossing the vertical axis (e.g., at 40 years). In a histogram, the mean is shown as a vertical line on the horizontal axis.

To assess variation in a case-value plot, one looks at the differences in height between the bars: the further the bars extend above and below the mean, the greater the variation. In a histogram, differences in bar height say something about frequencies, not about variation. There, variation is visible in the spread of the data along the horizontal axis. Suppose, for example, that all eight teachers are exactly 40 years old. In the case-value plot, all bars would then have the same height: there is no variation. If the same ages were displayed in a histogram, there would be one tall bar and all other intervals would be empty. This looks uneven, but here too there is no variation, because all teachers are the same age.

For case-value plots, the eye-tracking vignettes focus on the following systematic errors:

Systematic error	Definition
Overgeneralisation of a histogram approach	Incorrect assumption that a case-value plot can be interpreted like a histogram, based on shape or balance rather than on individual measured values.
Pairwise stack comparison	Judgement based on comparing corresponding stacks/bars one by one, rather than on the distribution as a whole.

4.4 Boxplots

Description

A boxplot is a graphical representation of the distribution of a quantitative variable, showing key statistical measures such as the median, range, and interquartile range (IQR). It is particularly useful for quickly comparing distributions. The boxplot consists of a rectangle (the "box") that represents the middle 50% of the data, between the first (lower) and third (upper) quartiles. The median, marked within the box, divides the data into two nearly equal parts. Lines, known as "whiskers," extend from the box: one from the minimum to the lower quartile, and the other from the upper quartile to the maximum (Figure 13).

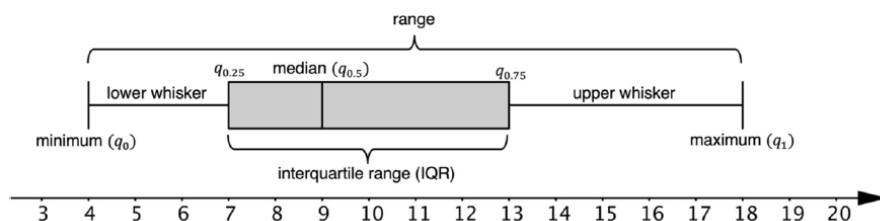


Figure 13. The boxplot representation showing key statistical measures (Abt et al., 2025).

One distinctive feature of the boxplot is that the representation juxtaposes measures of central tendency (like the median) and variability (range and IQR), which makes it useful for providing an overview of data. However, the boxplot can be challenging to interpret because it condenses multiple pieces of information into a single visualisation (Bakker et al., 2004).

Acquiring Conceptual Knowledge on Variability

Students need to interpret the box area as a measure of variability. However, classroom instruction tends to focus on procedural skills, such as drawing the boxplot, without developing a foundational understanding of variability (Edwards et al., 2017). This is unfortunate because the boxplot provides a powerful opportunity to acquire a first conceptualisation of variability. Conceptualising variability as deviation from a centre (e.g., standard deviation) ties variability to central tendency and increases cognitive load by requiring students to process multiple concepts at once. By contrast, conceptualising variability as distance between boundaries (e.g., range, interquartile range) is simpler and more intuitive.

The boxplot is ideal for teaching this approach, as it links the interquartile range (box area) to the well-known range and integrates both measures into a common conceptualisation of variability as distance between boundaries (Abt et al., 2024). When educators focus on conceptual rather than procedural knowledge, students not only interpret boxplots more accurately but also acquire conceptual knowledge about variability—one of the two core concepts in statistics alongside central tendency.

For boxplots, the eye-tracking vignettes focus on the following systematic errors:

Systematic error	Definition
Median bias	Judgement based on an incorrect measure of central tendency.
Area bias	Incorrect assumption that a longer box represents a higher frequency of observations.

5. Instructional Design of the Course (4C/ID)

The eye-tracking vignettes in the course have been designed using the Four-Component Instructional Design (4C/ID) model. The 4C/ID model was developed to guide the design of educational programmes aimed at teaching complex skills or professional competencies (Van Merriënboer et al., 2002; Van Merriënboer & Kirschner, 2018). It identifies four key components: learning tasks, supportive information, procedural information, and part-task practice.

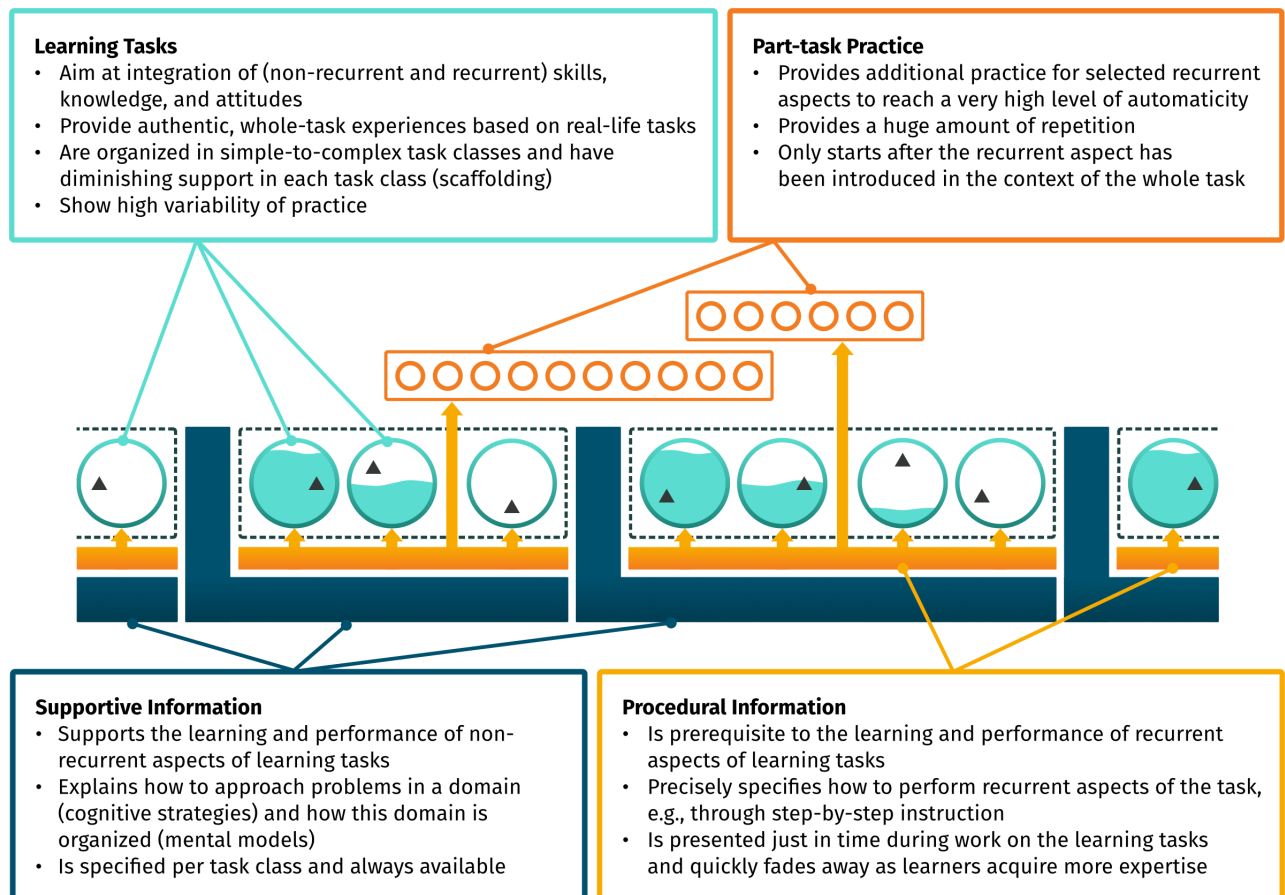


Figure 14. Graphical overview of the 4C/ID model

Note. A graphical view on the four components: (a) learning tasks, (b) supportive information, (c) procedural information, and (d) part-task practice, by Jeroen J. G. van Merriënboer, 2020, <https://www.4cid.org>. CC-BY-SA 4.0

Below, each 4C/ID component is described in relation to the design of the eye-tracking vignettes.

5.1 Learning Tasks

In each vignette, teachers are placed in a realistic classroom scenario focused on the interpretation of statistical graphs—dotplots, histograms, boxplots, and case-value plots. The vignettes are designed to reflect the kind of reasoning teachers are expected to perform in their own practice.

5.2 Supportive Information

Each vignette is accompanied by content knowledge related to statistical graph interpretation. This includes:

- definitions and purposes of the four graph types;
- examples showing how to interpret each graph type in context;
- brief explanations of key features and what to attend to when reading each graph.

This information helps participants activate relevant knowledge while working on the eye-tracking vignettes.

5.3 Procedural Information

Participants are introduced to different types of eye-tracking visualisations—gaze videos, heatmaps, and gaze plots—as procedural information. Descriptions of these data types are provided just in time: when eye-tracking data appears in a vignette, participants can access the relevant explanation via a button. This design provides guidance exactly when it is needed, without overloading participants in advance.

5.4 Part-Task Practice

At the end of each vignette, participants complete a targeted task that focuses on a key sub-skill related to the featured systematic error. These part-task activities typically involve:

- analysing visual attention patterns from eye-tracking data;
- identifying the systematic error evident in the vignette;
- reflecting on whether the same error might occur in a different context using the same graph type.

These short, focused exercises help participants consolidate core skills and support the transfer of these skills to real classroom situations.

6. Implementing the Course

This section provides guidance for teacher educators on how the course can be implemented in different instructional contexts. It outlines possible formats, the recommended sequence of activities, the role of synchronous group and individual work, and ways to support participants' reasoning while working with the eye-tracking vignettes.

6.1 Implementation Formats

The course is organised into five modules, with an introduction to eye-tracking and each focusing on one graph type: dotplots, histograms, case-value plots, and boxplots. It can be implemented in both pre-service teacher education and in-service professional development settings.

A blended format is recommended. In this format, synchronous group sessions are combined with individual work in MaMpf. Synchronous group sessions can be used to introduce the course approach, establish a shared language, work through selected vignettes collectively, and facilitate discussion. Individual work in MaMpf allows participants to complete the eye-tracking vignettes and related tasks at their own pace between sessions.

The course can be implemented in different ways, depending on the local context. For example, teacher educators may use it:

- as a full course in which all five modules are completed in sequence;
- as part of an existing course on statistics or mathematics didactics;
- as a short professional development programme;
- as a selective set of modules or vignettes chosen for a specific instructional focus.

The exact scheduling can be adapted to the available time and the needs of the participants. However, it is important that participants have sufficient time between sessions to engage with the vignettes, formulate their own interpretations, and revisit supporting materials when needed. Teachers typically need 15 to 30 minutes to work through one vignette.

6.2 Recommended Course Sequence

Although the course can be adapted flexibly, a consistent sequence is recommended to support participants' gradual development of analytical reasoning about students' graph interpretations.

The course begins with an introductory session, facilitated using slides 1–10 of the Teaching Modules deck. This session opens the course with a motivating question, introduces the concept of eye-tracking vignettes, and orients participants to MaMpf.

After the introductory session, each module can follow the same basic cycle:

1. **Module introduction.** The teacher educator briefly introduces the graph type and the analytical focus of the module in a synchronous group session, using the relevant slides from the Teaching Modules deck.
2. **Individual work in MaMpf.** Participants complete the eye-tracking vignettes and related tasks individually. They analyse students' responses and gaze data, formulate

- hypotheses about students' approaches, and identify possible systematic errors and underlying conceptual difficulties.
3. **Synchronous group discussion.** After the individual work, participants come together to discuss selected vignettes. The teacher educator uses the discussion slides in the Teaching Modules deck to structure the conversation.
 4. **Reflection and transfer.** Participants reflect on what the vignette analysis reveals about students' interpretations of statistical graphs and how these insights may inform their own teaching practice.

The sequencing of synchronous group sessions and individual work can also vary. Each module may be introduced in a dedicated synchronous group session, followed by individual work in MaMpf and a group discussion in the next session. Another possibility is that participants first complete several or all modules independently in MaMpf and then come together for a series of group discussions.

Where time is limited, teacher educators may also select individual vignettes from different modules and combine them around a shared analytical focus. For example, vignettes involving a similar systematic error across two different graph types can be discussed together to support comparison and transfer.

6.3 Facilitating Synchronous Group Meetings

Synchronous group meetings serve to introduce the approach, establish a shared language, and model the kind of reasoning expected in the course. These sessions are structured using the Teaching Modules slide deck.

Typical aims of synchronous group meetings include:

- introducing the purpose and structure of the course in relation to participants' own teaching practice;
- introducing key concepts such as systematic errors and the role of eye-tracking evidence in understanding students' graph interpretations;
- familiarising participants with the vignette format and the eye-tracking visualisations available in MaMpf;
- modelling how to formulate hypotheses about students' approaches;
- supporting participants in justifying their interpretations using evidence from student responses, task features, and eye-tracking visualisations.

During discussions, teacher educators can encourage participants to articulate their reasoning and to refer explicitly to observable evidence. For example, they may ask: *"Which aspect of the graph, the student's response, or the gaze pattern supports this interpretation?"*

The goal is not necessarily to arrive at a single correct answer. Rather, the aim is to develop participants' ability to ground their interpretations in evidence and to reason carefully about possible links between students' observable behaviour and their underlying conceptual difficulties.

The theoretical frameworks presented in this manual can be used as lenses for discussion. For example, teacher educators may invite participants to consider whether an observed response reflects a systematic error, an underlying conceptual difficulty, or both. Similarly, the distinction between local and global perspectives on graph interpretation can help participants discuss what students attend to when interpreting statistical graphs.

6.4 Working Through a First Vignette Together

Before participants begin working independently in MaMpf, it is recommended to work through one vignette together in a synchronous group session. This helps participants understand the vignette format and models the reasoning process expected throughout the course.

A suitable starting point is an example vignette accessible in MaMpf. During the collective session, teacher educators can guide participants through each step of the vignette structure, as described in Section 3.3. This may include:

- solving the statistical task themselves;
- examining the student's response;
- analysing the student's gaze data;
- formulating a hypothesis about the student's approach;
- comparing this hypothesis with the student's explanation;
- identifying possible systematic errors and underlying conceptual difficulties.

This collective analysis should be used to make the reasoning process explicit. Teacher educators can think aloud, ask participants to justify their interpretations, and show how different sources of evidence can be combined.

After working through the vignette, the corresponding discussion slide in the Teaching Modules deck can be used to structure a brief group reflection before participants continue independently in MaMpf.

6.5 Guiding Individual Work in MaMpf

In the individual part of the course, participants work with the eye-tracking vignettes in MaMpf. MaMpf is a learning environment in which participants complete the eye-tracking vignettes.

Teacher educators can support this process by:

- indicating which vignettes belong to each module;
- clarifying which vignettes participants are expected to complete before the next synchronous group meeting;
- providing general guidance on how to approach the vignettes;
- encouraging participants to formulate and refine hypotheses about students' approaches;
- asking participants to note the evidence they use when identifying systematic errors and underlying conceptual difficulties.

6.6 Discussion Slides, Guiding Questions, and Reflection

Each module in the Teaching Modules deck includes one or more discussion slides. These slides are intended to be used during synchronous group meetings after participants have worked through the eye-tracking vignettes in MaMpf. They are not meant as separate theoretical input, but as tools for structuring collective reasoning and reflection.

The discussion slides each contain notes for teacher educators, accessible in the notes panel of the slide deck. These notes include guiding questions tailored to the specific vignette and systematic error addressed in that slide. The questions are differentiated across modules to ensure that discussions address a range of aspects over the course of the programme.

The guiding questions can serve different purposes, depending on the module and discussion slide:

- **Conceptualising the error.** Some questions invite participants to define and discuss the nature of the systematic error: what it reveals about the student's understanding and how it differs from related errors.
- **Reading gaze data.** Some questions focus on the eye-tracking evidence: what specific gaze patterns indicate, and how visual behaviour can be used to support or refine a hypothesis about a student's approach.
- **Comparing across graph types.** Some questions ask participants to consider how the same or similar errors may appear across different graph types, supporting transfer of the analytical skills developed in the course.
- **Connecting to teaching practice.** Some questions invite participants to consider what an observed error implies for their own classroom practice: what they would ask a student, how they might design a follow-up task, or what the error suggests about instructional priorities.

Example guiding questions from the course include:

- What does this error reveal about the student's understanding of frequency versus data value?
- How does the gaze pattern help you distinguish this error from a random mistake?
- How is the sum of frequencies error different from frequency–value confusion? What does each reveal about the student's understanding?
- What would you learn about a student's understanding if they made this error on this task?
- What similarities can you identify in the student's approach across two different graph types?

Not all of these aspects need to be addressed in every discussion. Teacher educators can select the guiding questions most relevant to the current session focus, the selected vignettes, and where participants are in the course — for example, focusing on error conceptualisation in earlier sessions and connecting to teaching practice in later ones.

Reflection does not need to be limited to the discussion slides. Different aspects can be emphasised at different points in the course, and teacher educators can encourage participants to relate their observations to key statistical ideas such as distribution, centre, variability, frequency, and data values. After participants have completed one or more modules, this reflection can be extended through follow-up activities such as:

- identifying recurring patterns in students' reasoning across several vignettes;
- discussing how observed patterns relate to participants' own teaching experiences;
- formulating possible teacher responses to a student's interpretation;
- designing a follow-up task that could help address an observed difficulty;
- reflecting on how the eye-tracking data changed, confirmed, or refined participants' interpretations.

In this way, the course supports not only the analysis of specific vignette cases, but also the development of a broader perspective on students' approaches to statistical graphs. The aim is that participants can transfer this perspective to their own classroom practice when identifying and responding to students' statistical interpretations.

Project Information and Credits

1. About the Eye-Teach-Stats Project

Eye-Teach-Stats is an Erasmus+ project (2023-1-DE03-KA220-SCH-000158223) supporting pre- and in-service mathematics teachers in recognising and responding to students' systematic errors in statistical graph interpretation.

Aim: To support these teachers in (further) developing their statistical pedagogical content knowledge (PCK) and content knowledge (CK) — knowledge required to prepare students to become data-literate citizens — by providing opportunities to (1) diagnose and (2) address students' statistical (mis)interpretations, and (3) overcome their own statistical difficulties.

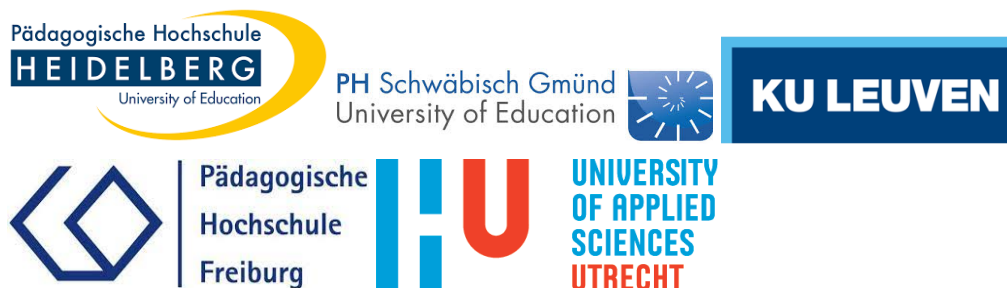
Target group: Pre-service teachers, in-service teachers, and student teachers of mathematics, as well as the teacher educators who train them.

Format: A five-module training course — eye-tracking information, dotplots, histograms, case-value plots, and boxplots — disseminated in English, German, and Dutch.

Duration: 1 September 2023 – 31 August 2026

More information: www.eye-teach-stats.eu

2. Project Partners



3. Authors

The Eye-Teach-Stats course materials were developed collaboratively by members of the project consortium:

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4. Funding, Disclaimer and Licence

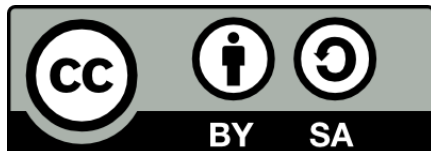
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